# AN ANALYSIS OF DYNAMIC ELEMENT MATCHING TECHNIQUES IN SIGMA-DELTA MODULATION

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## ABSTRACT

A mathematical model of mismatch noise in an oversampled DAC is established for two important dynamic element matching techniques. The noise shaping of the data weighted averaging method is proven to be first order. Analytical predictions of converter resolution can be made from array size, mismatch variance and oversampling ratio.

## 1. INTRODUCTION

Sigma-delta modulation has become the preferred technique for high-resolution data conversion [1]. Means of improving the performance of the converters are continually being sought. One promising avenue currently being explored by many researchers is the extension of traditional single threshold sigma-delta modulation to multibit quantization [2]. Significant advantages are to be obtained such as higher bandwidth and lower power consumption. Multibit sigma-delta modulation is particularily appropriate in the case of high resolution converters where the ıarge capacitors necessary for low thermal noise can be conveniently divided into smaller ones without area overhead.

The advantages of multibit quantization have been known for a long time. One major drawback has prevented its widespread application : converter linearity is severely limited by the matching of elements. Process lithography provides elements with a typical matching of 0.1-0.5% corresponding to a resolution of 8-10bits. Higher resolutions can be obtained by laser trimming or digital calibration techniques [3-4]. However, an attractive solution due to its simplicity and cost-effectiveness is *dynamic element matching* (DEM). The aim of DEM techniques is to modulate mismatch errors away from signal frequencies in order to remove them by filtering. An algorithm selects elements for each conversion such that they are used equally often and no single element can lead to the accumulation of a large linearity error.

Various DEM techniques have now been presented in the literature [5-9]. Up to now, the principal tool in the study of their mismatch noise has been system simulation. This paper provides an analytical model of the structure of mismatch noise. It is thereby possible to make theoretical predictions of converter resolution as a function of oversampling ratio, matching and number of elements. A detailed analysis of two typical techniques, *random selection* [5] and *data weighted averaging* [8-9] is presented. The postulated first order noise shaping of the data weighted averaging technique is confirmed analytically and extension to higher order noise shaping is foreseen [10].

The following treatment is based only on oversampled digital to analog converters (DAC's). However it should be pointed out that a major area of application of the results is in the DAC used in the feedback path of sigma delta analog to digital converters (ADC's).

## 2. DEFINITIONS

Most D/A converters are made up of a number N of nominally identical cells, such as current sources or capacitors. The D/A conversion is then realized by selecting a number of cells corresponding to the input code and by adding the contribution of the selected cells in order to generate an analog output voltage or current. Particular examples are D/A converters based on binary weighted capacitors. These converters are realized as an array of elementary identical capacitors, since, for bettı matching, each capacitor of weight $2^k$ is composed of a parallel combination of $2^k$ identical capacitors. As various elementary cells are then controlled by a same signal, the number of control lines for the whole array varies only with the logarithm in base 2 of the number of capacitors.

More generally, we assume here after that the N elementary cells of the D/A converter can be selected independently of each other. Let us denote by $w_i$ the weight of the i-th cell relative to its nominal value and by $d_i$ its control signal ($d_i$=1 when the cell is selected, otherwise $d_i$=0). The analog output signal y from the DAC can then be written as :

$$y = \sum_{i=0}^{N-1} d_i \cdot w_i \qquad (2.1)$$

where the number of selected cells is simply the input code x of the DAC :

$$x = \sum_{i=0}^{N-1} d_i \qquad (2.2)$$

Without any mismatch, the weight $w_i$ of each cell would be equal to its nominal value assumed to be 1, and hence y=x, meaning that the analog output is proportional to the input code, as in ideal case. However, due to mismatches, each weight $w_i$ deviates from its nominal value 1. Let us denote $w_{mean}$ as the average weight for the cells of the array :

$$w_{mean} = \frac{1}{N} \cdot \sum_{i=0}^{N-1} w_i \qquad (2.3)$$

With (2.2), the output code (2.1) can then be rewritten as

$$y = w_{mean} \cdot x + y_{mis} \qquad (2.4)$$

with

$$y_{mis} = \sum_{i=0}^{N-1} d_i \cdot (w_i - w_{mean}) \qquad (2.5)$$

being the error due to mismatch. This noise term will be evaluated for two different algorthms of dynamic element matching, which are random selection and cyclic selection

## 3. RANDOM SELECTION

This technique was proposed by Carley [5]. A number of cells corresponding to the input code is selected through a Butterfly decoder controlled by a pseudo random number generator. To simplify the evaluation of the random selection, we make the following hypotheses :

1) all the combinations of selected cells have the same probability.
2) the weights $w_i$ are random variables with expectation $E\{w_i\}=1$ and variance $s_w^2$
3) the weghts $w_i$ and $w_j$ are independent variables for $i \neq j$

As all the cells have the same probability density function, the variance of the mismatch is independent on the selected cells. Hence :

$$E\left\{y_{mis}^2\right\} = E\left\{\left[\sum_{i=0}^{x-1} w_i - \frac{x}{N} \cdot \sum_{i=0}^{N-1} w_i\right]^2\right\} = N \cdot \frac{x}{N} \cdot \left(1 - \frac{x}{N}\right) \cdot \sigma_w^2 \qquad (3.1)$$

This formula shows that the power of the mismatch error with this algorithm is modulated by the value x of the input code. This noise power cancels at the extremities of the range, for x=0, as no cell is selected, and for x=N, as all the cells are selected. In both cases, no mismatch error is introduced. The noise power is maximum in the middle of the range, for x =N/2, when half of the cells are selected. The power is then equal to $N s_w^2/4$. Figure 1 shows the RMS value of the mismatch noise $y_{mis}$ as a function of the input code x. The simulation results are close to the value predicted by (3.1).

Assuming that the cells are selected independently of the cells from one sampling cycle to the other, the mismatch noise has a white spectrum between 0 and fs/2 with fs being the sampling frequency. Assuming that the converter is oversampled by a factor M, only 1/M of the mismatch noise power will then fall into the signal baseband. As the signal swing runs from 0 to N, the maximum resolution that can be obtained is given by :

$$resolution = \log 2\left(\frac{\sqrt{\frac{N \cdot M}{3}}}{\sigma_w}\right) \qquad [bits]$$

## 4. CYCLIC SELECTION

In sigma delta modulation, high resolution is obtained by combining oversampling together with noise shaping in order to modulate the quantization noise outside the baseband, so that most of the noise can further be eliminated by filtering. In fact, noise shaping techniques can also be combined together with dynamic element matching techniques in order to reduce the sensitivity to matching. In the particular case of first order low pass sigma delta modulation, this technique consists of accumulating the mismatch error and selecting the cells in such a way that the accumulated error remains as small as possible. Basically, this can be realized by counting the number of times each cell is selected and by selecting the cells in such a way that all cells are more or less equally used.

A very efficient implementation of such an algorithm is the cyclic cell selection, as proposed by [8], and also refered to as data weighted averaging [9]. This algorithm can be implemented by using a digital register ptr called pointer containing the address of one cell of the array, with 0<=ptr<N. At each clock cycle, this pointer is incremented modulo N by the input code :

$$ptr(k) = ptr(k-1) + x(k) \mod N \qquad (4.1)$$

The cells selected at time k are the cells labelled from ptr(k-1) to ptr(k) by increasing order, that is cells ptr(k-1), ptr(k-1)+1, ptr(k-1)+2,...ptr(k)-1 if ptr(k-1)<=ptr(k), otherwise ptr(k-1), ptr(k-1)+1,...N-1, 0, 1, ... ptr(k)-1 if ptr(k-1)>ptr(k).

The mismatch errror at time k is then given by :

*if* $ptr(k) \geq ptr(k-1)$

$$y_{mis}(k) = \sum_{i=ptr(k-1)}^{ptr(k)-1} w_i - x(k) \cdot w_{mean}$$

*if* $ptr(k) < ptr(k-1)$

$$y_{mis}(k) = \sum_{i=ptr(k-1)}^{N-1} w_i + \sum_{i=0}^{ptr(k)-1} w_i - x(k) \cdot w_{mean}$$

(4.2)

In order to demonstrate that this mismatch error benefits from first order noise shaping, let us first remark that (4.2) can be written in the form :

$$y_{mis}(k) = IM(ptr(k)) - IM(ptr(k-1)) \qquad (4.3)$$

with IM(ptr) being a function of the pointer on the array called the Integral Mismatch function. This function is obtained by integrating the mismatches of cells along the array :

$$IM(ptr) = \sum_{i=0}^{ptr-1} (w_i - w_{mean}) + IM(0) = \sum_{i=0}^{ptr-1} w_i - ptr \cdot w_{mean} + IM(0) \qquad (4.4)$$

As $y_{mis}(k)$ is obtained by first order differentiation of the function IM(ptr(k)), it benefits from first order noise shaping. Indeed, (4.3) can be written in the Z-domain into the form :

$$Y_{mis}(Z) = (1 - Z^{-1}) \cdot IM(PTR(Z)) \qquad (4.5)$$

The value IM(0) of the integral mismatch function for ptr=0 is defined in order to cancel the average value of IM(ptr) :

$$\sum_{ptr=0}^{N-1} IM(ptr) = 0 \qquad (4.6)$$

Eliminating IM(0) from (4.4) by means of (4.6) gives :

$$IM(ptr) = \sum_{i=0}^{ptr-1} w_i + \frac{1}{N} \cdot \sum_{i=0}^{N-1} \left( i - \frac{N-1}{2} - ptr \right) \cdot w_i$$

(4.7)

Assuming ptr1<=ptr2, one obtains after some calculation :

$$E\{IM(ptr1) \cdot IM(ptr2)\} =$$

$$\frac{\sigma_w^2}{N^2} \cdot \left\{ \frac{N^3 - N}{12} - \frac{N}{2} \cdot (ptr2 - ptr1) \cdot [N - (ptr2 - ptr1)] \right\}$$

(4.8)

This expression will be used in order to compute the selfcorrelation function of the mismatch noise, which is given by :

$$\Psi(p) = E\{IM(ptr(k) \cdot IM(ptr(k+p)\} \qquad (4.9)$$

The spectrum of the mismatch noise will further be derived. These calculations will be done in two different cases : for a random input signal and for a DC input signal.

1) random input signal

In this case, we consider that all the input values of the input code from 0 to N-1 have the same probability equal to 1/N at each clock cycle. Under these conditions, one can easily show :

$$\Psi(k) = \frac{\sigma_w^2 \cdot (1 - 1/N)^2 \cdot N}{12} \quad for \quad k = 0$$
$$\Psi(k) = 0 \quad for \quad k \neq 0 \qquad (4.10)$$

This means that IM(ptr(k)) has a white spectrum and its total power is Y(0). The spectrum of $Y_{mis}$ can then be derived due to (4.5). Assuming that the DAC is oversampled by a factor M, the maximum achievable resolution is given by :

$$resolution = \log 2 \left( \frac{\sqrt{3 \cdot N \cdot M^3}}{\pi \cdot \sigma_w \cdot \left(1 - \frac{1}{N}\right)} \right) \quad [bits] \quad (4.11)$$

2) DC input signal

In this case, assuming x(k)=X, (4.1) gives :

$$ptr(k) - ptr(k+p) = p \cdot X \mod N \qquad (4.12)$$

Combining with (4.8), the selfcorrelation function (4.9) can be written as :

$$\Psi(p) = \frac{\sigma_w^2}{N^2} \cdot \left\{ \frac{N^3 - N}{12} - \frac{N}{2} \cdot \alpha(x) \cdot [N - \alpha(x)] \right\} \quad (4.13)$$

$$with \quad \alpha(x) = p \cdot X \mod N$$

The values of the selfcorrelation function can thus be obtained by sampling of a continuous time f(t) at the sampling rate fs=1/Ts. This function f(t) is periodic of period Tp=Ts·N/X :

$$\phi(t) = \frac{\sigma_w^2}{N^2} \cdot \left\{ \frac{N^3 - N}{12} - \frac{N}{2} \cdot \left( \frac{X \cdot t}{Ts} \right) \cdot \left[ N - \left( \frac{X \cdot t}{Ts} \right) \right] \right\}$$
$$= \frac{\sigma_w^2}{N^2} \cdot \left\{ \frac{N^3 - N}{12} - \frac{N^3}{2} \cdot \frac{t}{Tp} \cdot \left[ 1 - \frac{t}{Tp} \right] \right\}$$
$$for \quad 0 \leq t \leq Tp = \frac{N}{X} \cdot Ts$$
$$\phi(t) = \phi(t - k \cdot Tp)$$
$$for \quad k \cdot Tp \leq t \leq (k+1) \cdot Tp$$

(4.14)

As f(t) is periodic with period Tp and symetric with respect to t=0, it can be written as :

$$\phi(t) = A_0 + \sum_{i=1}^{\infty} A_i \cdot \cos(i \cdot 2\pi \cdot f \cdot Tp) \qquad (4.15)$$

with

$$A_0 = -\frac{\sigma_w^2}{12 \cdot N}, \qquad A_i = \frac{2 \cdot N \cdot \sigma_w^2}{(2\pi \cdot i)^2} \qquad (4.16)$$

Hence the spectrum of the continuous time signal f(t) is a sum of components of amplitude $A_i$ located at frequency ±i/Tp. By consequence, as Y(p) is obtained by sampling f(t) at frequency fs, its spectrum will be computed by folding all the components between -fs/2 and +fs/2. The spectrum of Y(p), which is the power density of the mismatch noise source IM(ptr(k)), is thus made of components of power $A_i$ located at frequency i·fp-fs·round(i·fp/fs) where round(x) represents the rounding of x to the closest integer. As fp is related to the DC signal level, the spectrum of the mismatch noise will also depend on the DC level. For values of DC level close to 1/2, 1/3, 2/3 ,1/4, 3/4,... of the range, a low order noise component of high power is folded back into the baseband, and the equivalent resolution is decreased, in a similar way as for the quantization noise with first order sigma delta modulation [11]. Figure 2 shows the simulated RMS noise as a function of the DC level, compared to the theoretical value corresponding to (4.11) for random input signals. This estimation corresponds to an average value over the whole DC range.

## CONCLUSIONS

The structure of DAC mismatch noise for DC signals has been analyzed and shown to contain peaks at certain input levels. The demonstration of first order noise shaping for the case of data weighted averaging method points the way to DEM techniques for higher order noise shaping. Analytical predictions of converter resolution can now usefully be made from array size, mismatch variance and oversampling ratios. The mathematical models show close agreement with computer simulation results.

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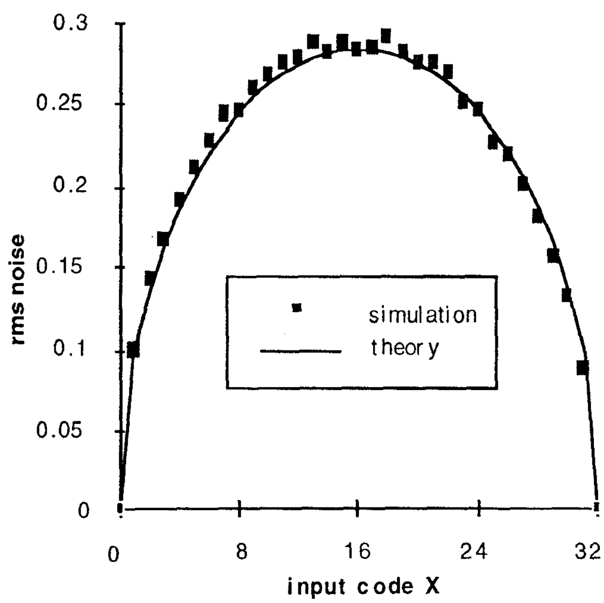


Fig. 1 : RMS mismatch noise as a function of the input code, for random cell selection

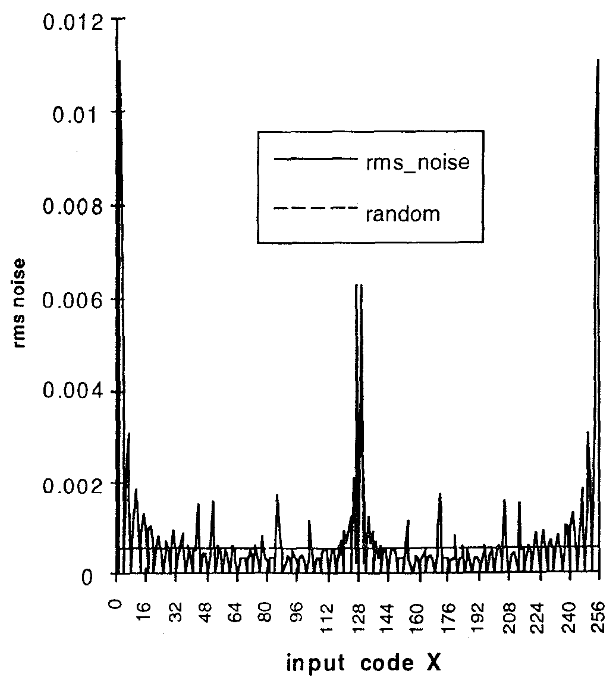


Fig. 2 : RMS mismatch noise as a function of input code level for cyclic method for a DAC with N=256 elements , 10% mismatch and oversampling M=128.